Connecting the Concepts

Algebraic and Graphical Methods

We have considered both algebraic and graphical methods of solving polynomial equations. It is important to understand and be able to use both methods. Some of the advantages and disadvantages of each method are given in the following table.

	Advantages	Disadvantages
Algebraic Method	• Can find exact answers • Works well when the polynomial is in factored form or can be readily factored • Can be used to find solutions that are not real numbers (see Chapter 8)	• Cannot be used if the polynomial is not factorable • Can be difficult to use if factorization is not readily apparent
Graphical Method	• Does not require the polynomial to be factored • Can visualize solutions	• Easy to miss solutions if an appropriate viewing window is not chosen • Gives approximations of solutions • For most graphing calculators, only real-number solutions can be found.

5.6 Exercise Set

FOR EXTRA HELP MyMathLab MathXL PRACTICE WATCH DOWNLOAD READ REVIEW

Concept Reinforcement *Identify each of the following as a perfect-square trinomial, a difference of two squares, a prime polynomial, or none of these.*

1. $9t^2 - 49$ Difference of two squares
2. $25x^2 - 20x + 4$ Perfect-square trinomial
3. $36x^2 - 12x + 1$ Perfect-square trinomial
4. $36a^2 - 25$ Difference of two squares
5. $4x^2 + 8x + 10$ None of these
6. $x^2 + 1$ Prime polynomial
7. $100y^2 + z^2$ Prime polynomial
8. $t^2 - 6t + 8$ None of these

Determine whether each of the following is a perfect-square trinomial.

9. $x^2 + 18x + 81$ Yes
10. $x^2 - 16x + 64$ Yes
11. $x^2 - 10x - 25$ No
12. $x^2 - 14x - 49$ No
13. $x^2 - 3x + 9$ No
14. $x^2 + 4x + 4$ Yes
15. $9x^2 + 25 - 30x$ Yes
16. $36x^2 + 16 - 24x$ No

Factor completely.

17. $t^2 + 6t + 9$ $(t + 3)^2$
18. $a^2 + 16a + 64$ $(a + 8)^2$
19. $a^2 - 14a + 49$ $(a - 7)^2$
20. $x^2 - 8x + 16$ $(x - 4)^2$
21. $4a^2 - 16a + 16$ $4(a - 2)^2$
22. $2a^2 + 8a + 8$ $2(a + 2)^2$
23. $1 - 2t + t^2$ $(t - 1)^2$, or $(1 - t)^2$
24. $1 + t^2 + 2t$ $(t + 1)^2$
25. $24a^2 + a^3 + 144a$ $a(a + 12)^2$
26. $-18y^2 + y^3 + 81y$ $y(y - 9)^2$
27. $20x^2 + 100x + 125$ $5(2x + 5)^2$
28. $32x^2 + 48x + 18$ $2(4x + 3)^2$
29. $1 + 8d^3 + 16d^6$ $(4d^3 + 1)^2$
30. $64 + 25y^8 - 80y^4$ $(5y^4 - 8)^2$, or $(8 - 5y^4)^2$

31. $-y^3 + 8y^2 - 16y$ $-y(y-4)^2$

32. $10a^2 - a^3 - 25a$ $-a(5-a)^2$, or $-a(a-5)^2$

33. $0.25x^2 + 0.30x + 0.09$ $(0.5x + 0.3)^2$

34. $0.04x^2 - 0.28x + 0.49$ $(0.2x - 0.7)^2$

35. $x^2 - 2xy + y^2$ $(x-y)^2$

36. $m^{10} + 2m^5n^5 + n^{10}$ $(m^5 + n^5)^2$

37. $25a^6 + 30a^3b^3 + 9b^6$ $(5a^3 + 3b^3)^2$

38. $49p^2 - 84pt + 36t^2$ $(7p - 6t)^2$

39. $5a^2 - 10ab + 5b^2$ $5(a-b)^2$

40. $4t^2 - 8tr + 4r^2$ $4(t-r)^2$

Determine whether each of the following is a difference of squares.

41. $x^2 - 100$ Yes

42. $x^2 + 49$ No

43. $n^4 + 1$ No

44. $n^4 - 81$ Yes

45. $-1 + 64t^2$ Yes

46. $-12 + 25t^2$ No

Factor completely. Remember to look first for a common factor. If a polynomial is prime, state this.

47. $y^2 - 100$ $(y+10)(y-10)$

48. $x^2 - 16$ $(x+4)(x-4)$

49. $m^2 - 64$ $(m+8)(m-8)$

50. $q^2 + 1$ Prime

51. $-49 + t^2$ ⊡

52. $-64 + m^2$ ⊡

53. $8x^2 - 8y^2$ $8(x+y)(x-y)$

54. $6x^2 - 6y^2$ $6(x+y)(x-y)$

55. $-80a^6 + 45$ $-5(4a^3+3)(4a^3-3)$

56. $-81x^7 + 16x$ $-x(9x^3+4)(9x^3-4)$

57. $49a^4 + 100$ Prime

58. $9x^4 - 25x^2$ $x^2(3x+5)(3x-5)$

59. $t^4 - 1$ $(t^2+1)(t+1)(t-1)$

60. $x^4 - 16$ $(x^2+4)(x+2)(x-2)$

61. $9a^4 - 25a^2b^4$ ⊡

62. $16x^6 - 121x^2y^4$ ⊡

63. $16x^4 - y^4$ $(4x^2+y^2)(2x+y)(2x-y)$

64. $2a^4 - 32y^8$ ⊡

65. $\frac{1}{49} - x^2$ $\left(\frac{1}{7}+x\right)\left(\frac{1}{7}-x\right)$

66. $\frac{1}{16} - y^2$ $\left(\frac{1}{4}+y\right)\left(\frac{1}{4}-y\right)$

67. $(a+b)^2 - 9$ $(a+b+3)(a+b-3)$

68. $(p+q)^2 - 25$ $(p+q+5)(p+q-5)$

69. $x^2 - 6x + 9 - y^2$ $(x-3+y)(x-3-y)$

70. $a^2 - 8a + 16 - b^2$ $(a-4+b)(a-4-b)$

71. $t^3 + 8t^2 - t - 8$ $(t+8)(t+1)(t-1)$

72. $x^3 - 7x^2 - 4x + 28$ $(x-7)(x+2)(x-2)$

73. $r^3 - 3r^2 - 9r + 27$ $(r-3)^2(r+3)$

74. $t^3 + 2t^2 - 4t - 8$ ⊡

75. $m^2 - 2mn + n^2 - 25$ $(m-n+5)(m-n-5)$

76. $x^2 + 2xy + y^2 - 9$ $(x+y+3)(x+y-3)$

77. $36 - (x+y)^2$ $(6+x+y)(6-x-y)$

78. $49 - (a+b)^2$ $(7+a+b)(7-a-b)$

Aha! **79.** $16 - a^2 - 2ab - b^2$ $(4+a+b)(4-a-b)$

80. $9 - x^2 - 2xy - y^2$ $(3+x+y)(3-x-y)$

81. $a^3 - ab^2 - 2a^2 + 2b^2$ $(a-2)(a+b)(a-b)$

82. $p^2q - 25q + 3p^2 - 75$ $(p+5)(p-5)(q+3)$

Solve. Round any irrational solutions to the nearest thousandth.

83. $a^2 + 1 = 2a$ 1

84. $r^2 + 16 = 8r$ 4

85. $2x^2 - 24x + 72 = 0$ 6

86. $-t^2 - 16t - 64 = 0$ -8

⊡ Answers to Exercises 51, 52, 61, 62, 64, and 74 are on p. IA-13.

87. $x^2 - 9 = 0$ $-3, 3$

88. $r^2 - 64 = 0$ $-8, 8$

89. $a^2 = \frac{1}{25}$ $-\frac{1}{5}, \frac{1}{5}$

90. $x^2 = \frac{1}{100}$ $-\frac{1}{10}, \frac{1}{10}$

91. $8x^3 + 1 = 4x^2 + 2x$ $-\frac{1}{2}, \frac{1}{2}$

92. $27x^3 + 18x^2 = 12x + 8$ $-\frac{2}{3}, \frac{2}{3}$

93. $x^3 + 3 = 3x^2 + x$ $-1, 1, 3$

94. $x^3 + x^2 = 16x + 16$ $-4, -1, 4$

95. $x^2 - 3x - 7 = 0$ $-1.541, 4.541$

96. $x^2 - 5x + 1 = 0$ 0.209, 4.791

97. $2x^2 + 8x + 1 = 0$ $-3.871, -0.129$

98. $3x^2 + x - 1 = 0$ $-0.768, 0.434$

99. $x^3 + 3x^2 + x - 1 = 0$ $-2.414, -1, 0.414$

100. $x^3 + x^2 + x - 1 = 0$ 0.544

101. Let $f(x) = x^2 - 12x$. Find a such that $f(a) = -36$. 6

102. Let $g(x) = x^2$. Find a such that $g(a) = 144$. $-12, 12$

Find the zeros of each function.

103. $f(x) = x^2 - 16$ $-4, 4$

104. $f(x) = x^2 - 8x + 16$ 4

105. $f(x) = 2x^2 + 4x + 2$ -1

106. $f(x) = 3x^2 - 27$ $-3, 3$

107. $f(x) = x^3 - 2x^2 - x + 2$ $-1, 1, 2$

108. $f(x) = x^3 + x^2 - 4x - 4$ $-2, -1, 2$

TW **109.** Explain in your own words how to determine whether a polynomial is a perfect-square trinomial.

TW **110.** Explain in your own words how to determine whether a polynomial is a difference of squares.

SKILL REVIEW

To prepare for Section 5.7, review the product and power rules for exponents and multiplication of polynomials (Sections 1.4 and 5.2).

Simplify. [1.4]

111. $(2x^2y^4)^3$ $8x^6y^{12}$

112. $(-5x^2y)^3$ $-125x^6y^3$

Multiply. [5.2]

113. $(x+1)(x+1)(x+1)$ $x^3 + 3x^2 + 3x + 1$

114. $(x-1)^3$ $x^3 - 3x^2 + 3x - 1$

115. $(m+n)^3$ $m^3 + 3m^2n + 3mn^2 + n^3$

116. $(m-n)^3$ $m^3 - 3m^2n + 3mn^2 - n^3$

SYNTHESIS

TW **117.** Without finding the entire factorization, determine the number of factors of $x^{256} - 1$. Explain how you arrived at your answer.

TW **118.** Akio concludes that since $x^2 - 9 = (x-3)(x+3)$, it must follow that $x^2 + 9 = (x+3)(x-3)$. What mistake(s) is he making?